MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

ETN 2036 – INTRODUCTION TO DIGITAL SIGNAL PROCESSING

(All sections / Groups)

3rd MARCH 2016
2.30 pm – 4.30 pm
(2 Hours)

INSTRUCTION TO STUDENTS

1. This question paper consists of 5 pages (including cover page) with 4 questions and 7-page list of formulas given in the appendix.

2. Attempt all **FOUR** questions. The distribution of the marks for each question is given. The overall total marks is 60.

3. Please write all your answers in the answer Booklet provided.

QUESTION 1

a) State TWO advantages and disadvantages of digital signal processing (DSP) systems over conventional analogue systems. [2 marks]

b) Determine whether or not the following systems are: (1) stable, and (2) linear. Justify your answer.

i) $y[n] = \sum_{k=n-2}^{n+2} x[k]$ [4 marks]

ii) $y[n] = x[n] + 3u[n + 1]$ [4 marks]

c) Evaluate the linear convolution sum of the following two sequences. You may use any method you prefer.

$$x_1[n] = [1, \underline{2}, 4], \quad x_2[n] = [2, \underline{-3}, 2]$$

The underlined value corresponds to $n = 0$. Leave your answer in a similar format. [3 marks]

d) Determine whether the following input signals to a DSP system will suffer from aliasing problem by using the given sampling period T.

i) $x(t) = 2\cos(10\pi t)$, $T = 0.1$ s [1 mark]

ii) $x(t) = 8\cos(15\pi t)$, $T = 0.2$ s [1 mark]

Continued..

QUESTION 2

a) A system is described by the transfer function, $H(z)=\frac{z}{3z^2-4z+1}$.

Determine its impulse response for the following regions of convergence (ROC). State with justification whether or not the system is realizable for each ROC.

i) $|z|>1$ [3.5 marks]

ii) $\frac{1}{3}<|z|<1$ [3.5 marks]

b) It is desired to estimate the frequency spectrum of an electrocardiogram (ECG) signal with the use of Fast Fourier Transform (FFT) radix 2 method. The ECG signal shows the variation of the heart electrical activity over time. The analog signal is sampled with the sampling rate of 10000 Hz and a frequency resolution of 1 Hz is needed for spectrum measurement. Compute the minimum number of samples that need to be collected for the signal spectrum measurement scheme.

[2 Marks]

c) Apply 4-point Discrete Fourier Transform (DFT) on the sequence $x[n]$ = [1 1 1 0] to obtain the DFT coefficients $X[k]$. The underlined sample denote $x[0]$. Given that the sampling frequency used is 10,000 Hz, plot the magnitude response $|X(k)|$ vs frequency in Hz. All the four points within the frequency range [0 , 10000 Hz] must be plotted. Label all the DFT coefficients magnitude and its corresponding discreet frequency.

[6 Marks]

Continued..

QUESTION 3

a) i) Explain the rationale of truncating the impulse response of an ideal finite impulse response (FIR) digital filter using a window function. Briefly describe the effects caused by this truncation and the method to minimize them. [3 marks]

ii) An acoustic signal recorded digitally at a sampling frequency of 1000 Hz contains useful content from 0 to 300 Hz. The recording environment introduces noise from 400 to 500 Hz into the signal. Design a digital FIR low-pass filter using a fixed window to attenuate the noise by at least 52 dB without affecting the useful content by more than 3 dB. [7 marks]

b) i) Explain the four major steps involved in designing an IIR filter with the Bilinear transformation design method.

[2 marks]

ii) Design a low pass digital Butterworth filter using the bilinear transform method with minimum filter order and satisfying the following specifications:

Passband frequency range	: 0 - 1 kHz
The attenuation at passband edge frequency	: 3 dB
Stopband edge frequency	: 1.5 kHz
Minimum stopband attenuation	: 10 dB
Sampling frequency	: 8 kHz.

You will need to compute the transfer function for the prototype analogue Butterworth filter and the transfer function for the digital filter.

[8 marks]

Continued ...

QUESTION 4

a) Implement the structure of an Infinite Impulse Response (IIR) filter with the following 2nd order transfer function $H(z) = \frac{0.4\,(1-z^{-1})}{(1+0.4z^{-1})(1+0.8z^{-1})}$. Use the following specified structure.

i) Direct form 1

[2 marks]

ii) Direct form 2 (canonical structure)

[2 marks]

iii) Two different cascade structure with first order section.

[3 marks]

iv) Parallel structure with first order section.

[3 marks]

End of paper

APPENDIX

SUMMARY OF USEFUL TABLES

DISCRETE-TIME FOURIER TRANSFORM PAIRS

Sequence	Discrete-Time Fourier Transform
$\delta[n]$	1
$\delta[n-n_o]$	$e^{-j\omega n_o}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega} + r^2 e^{-j\omega})}$
$\frac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \omega_c < \lvert\omega\rvert < \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & otherwise \end{cases}$	$\frac{\sin[\omega(M+1)/2)]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{n=-\infty}^{\omega} 2\pi\delta(\omega-\omega_0+2\pi k)$
$\cos(\omega_0 n+\phi)$	$\sum_{n=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega-\omega_0+2\pi k) + \pi e^{-j\phi}\delta(\omega+\omega_0+2\pi k)]$

SOME COMMON z-TRANSFORM PAIRS

The z-transform

Properties of the z-transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n]+by[n]$	$aX(z)+bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n-m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n]*y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common z-transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n-n_0]$	z^{-n_0}	Possibly $\forall z$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$

SOME COMMON WINDOWS FOR FIR FILTER DESIGN

Ideal lowpass filter, $n = 0, 1, 2, \ldots, N$ (filter order given by N)

$$h[n] = \frac{\sin\left(\omega_c\left(n-\frac{N}{2}\right)\right)}{\left(n-\frac{N}{2}\right)\pi} = \frac{\omega_c}{\pi}\,\mathrm{sinc}\left(\frac{\omega_c\left(n-\frac{N}{2}\right)}{\pi}\right)$$

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Ideal bandpass filter, $n = 0, 1, 2, \ldots, N$ (filter order given by N)

$$h[n] = \frac{\omega_2}{\pi}\,\mathrm{sinc}\left(\frac{\omega_2\left(n-\frac{N}{2}\right)}{\pi}\right) - \frac{\omega_1}{\pi}\,\mathrm{sinc}\left(\frac{\omega_1\left(n-\frac{N}{2}\right)}{\pi}\right)$$

Rectangular	$w[n] = \begin{cases} 1 & 0 \le n \le N \\ 0 & else \end{cases}$
Hanning	$w[n] = \begin{cases} 0.5 - 0.5\cos\left(\frac{2n\pi}{N}\right) & 0 \le n \le N \\ 0 & else \end{cases}$
Hamming	$w[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2n\pi}{N}\right) & 0 \le n \le N \\ 0 & else \end{cases}$
Blackman	$w[n] = \begin{cases} 0.42 - 0.5\cos\left(\frac{2n\pi}{N}\right) + 0.08\cos\left(\frac{4n\pi}{N}\right) & 0 \le n \le N \\ 0 & else \end{cases}$
Kaiser	$w[n] = \frac{I_0\left(\beta\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0(\beta)} \quad 0 \le n \le N \quad \alpha = N/2$ where $\beta = \begin{cases} 0.1102(A-8.7), & A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & 21 \le A \le 50 \\ 0, & A < 21 \end{cases}$ $N = \begin{cases} (A-7.95)/(2.285\Delta\omega), & A \ge 21 \\ 5.655/\Delta\omega, & A < 21 \end{cases}$

COMPARISON OF COMMONLY USED WINDOWS

Type of Window (Filter length = N+1)	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Stopband Attenuation, $20\log_{10}\delta_s$ (dB)	Transition Width ($\Delta\omega$, $\lvert\omega_s - \omega_p\rvert$)	Transition Width (Δf, $\frac{\lvert f_s - f_p \rvert}{f_{sampling}}$)
Rectangular	-13	$4\pi/(N+1)$	-21	$1.8\pi/N$	$0.9/N$
Hanning	-31	$8\pi/N$	-44	$6.2\pi/N$	$3.1/N$
Hamming	-41	$8\pi/N$	-53	$6.6\pi/N$	$3.3/N$
Blackman	-57	$12\pi/N$	-74	$11\pi/N$	$5.5/N$

IIR FILTER DESIGN

THE COEFFICIENTS IN THE SYSTEM FUNCTION OF A NORMALIZED BUTTERWORTH LOW PASS FILTER (Ω_c=1) FOR ORDERS $1 \le N \le 8$

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.0000							
2	1.4142	1.0000						
3	2.0000	2.0000	1.0000					
4	2.6131	3.4142	2.6131	1.0000				
5	3.2361	5.2361	5.2361	3.2361	1.0000			
6	3.8637	7.4641	9.1416	7.4641	3.8637	1.0000		
7	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1.0000	
8	5.1258	13.1371	21.8462	25.6884	21.8462	13.1372	5.1258	1.0000

CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \qquad |a| < 1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

PARAMETERS OF ANALOG FILTER PROTOTYPES

Discrimination factor $d = \left[\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1} \right]^{1/2}$

Selectivity factor $k = \frac{\Omega_p}{\Omega_s}$

Bilinear transformation

$\Omega_p = \frac{2}{T_s} \tan\left(\frac{\omega_p}{2}\right)$

$\Omega_s = \frac{2}{T_s} \tan\left(\frac{\omega_s}{2}\right)$

Design	Filter order
Butterworth	$N \geq \frac{\log d}{\log k}$
Chebyshev I, II	$N \geq \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$
Elliptic	$N \geq \frac{\log(16/d^2)}{\log(2/u)}$
	$u = \frac{1-(1-k^2)^{0.25}}{1+(1-k^2)^{0.25}}$

Frequency transformations

Target class	Transformation	Edge frequencies of target class
Highpass	$s \to \frac{\Omega_p \Omega'_p}{s}$	Ω'_p
Bandpass	$s \to \Omega_p \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}$	Ω_l, Ω_u
Bandstop	$s \to \Omega_p \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$	Ω_l, Ω_u

Impulse invariance transformation

$$H_a(s) = \sum_{k=0}^{p-1} \frac{A_k}{s - s_k} \longrightarrow H(z) = \sum_{k=0}^{p-1} \frac{T_s A_k}{1 - e^{s_k T_s} z^{-1}}$$

Bilinear transformation

$$H(z) = H_a(s)\Big|_{s = \frac{2}{T_s} \frac{1-z^{-1}}{1+z^{-1}}}$$

$$\Omega = 2 \tan(w/2)/T_s$$

$$\omega = \frac{2\pi f}{f_{samp}} \quad f_{samp} = \frac{1}{T_s}$$

Discrete-time Fourier Analysis

The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

Properties of the DFT

Property	$x[n]$	$X[k]$
Linearity	$A_1 x_1[n] + A_2 x_2[n]$	$A_1 X_1[k] + A_2 X_2[k]$
Time shifting	$x[\langle n - n_0 \rangle_N]$	$X[k] W_N^{kn_0}$
Frequency shifting	$x[n] W_N^{-k_0 n}$	$X[\langle k - k_0 \rangle_N]$
Time reversal	$x[\langle -n \rangle_N]$	$X[\langle -k \rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k \rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k] Y[k]$
Modulation	$N x[n] y[n]$	$X[k] \circledast Y[k]$

The decimation-in-time FFT